Representing BRDF by Wavelet Transformation of Pair-Copula Constructions

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Table of Contents

1. Introduction
   - Definitions of BRDFs
   - Properties of Proposed BRDF Representation
   - Our Proposed BRDF Representation
2. Previous Works
3. Copula Distributions
   - Pair-Copula Constructions
4. BRDF Representation Using Pair-Copula Constructions
   - MIT MERL BRDF Dataset
   - Our BRDF Representation
   - Some Empirical Results
5. Our Estimation Procedure
6. Importance Sampling
7. Results
8. Conclusions and Future Work
9. Questions
10. Thank You
11. References

Introduction

- Modeling the surface reflectance of light is an important issue in computer graphics.

Isotropic Reflection

Anisotropic Reflection

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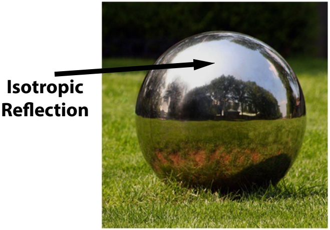
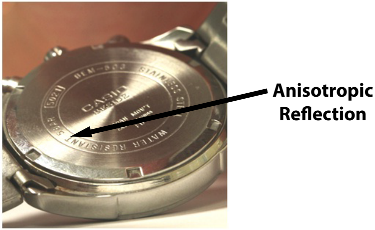
- Expressing the surface reflectance by a mathematical model has been studied extensively.





Introduction

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- Expressing the surface reflectance by a mathematical model has been studied extensively.
- BRDFs are commonly used as mathematical models to describe the surface reflectance.

Isotropic Reflection

Anisotropic Reflection

- BRDF was first formulated by Nicodemus et al. [12] as

$$\rho(\vec{\omega}_i, \vec{\omega}_o) = \frac{dL_o(\vec{\omega}_o)}{L_i(\vec{\omega}_i)\cos\theta_i d\vec{\omega}_i}, \quad (1)$$

where $\rho(\vec{\omega}_i, \vec{\omega}_o)$ is the BRDF, L_i and L_o are the incident and reflected radiance, respectively, $(\vec{\omega}_i, \vec{\omega}_o) = \{(\theta_i, \phi_i), (\theta_o, \phi_o)\}$ are the corresponding incoming and outgoing vectors, $d\vec{\omega}_i$ is the differential solid angle in the ω_i direction.

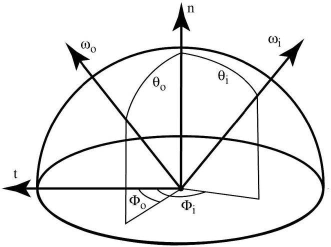

Table of Contents

1. Introduction
    - Definitions of BRDFs
    - Properties of Proposed BRDF Representation
    - Our Proposed BRDF Representation
2. Previous Works
3. Copula Distributions
    - Pair-Copula Constructions
4. BRDF Representation Using Pair-Copula Constructions
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6. Importance Sampling
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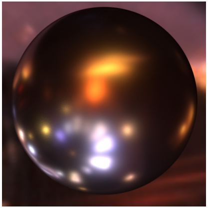
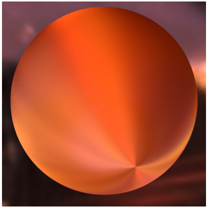


Nickel

Isotropic BRDFs



Yellow Satin

Table of Contents

1. Introduction
   - Definitions of BRDFs
   - Properties of Proposed BRDF Representation
   - Our Proposed BRDF Representation
2. Previous Works
3. Copula Distributions
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6. Importance Sampling
7. Results
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9. Questions
10. Thank You
11. References

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- Since our proposed model is based on the parametrization by Rusinkiewicz [16], we enforce our system with the following translation to ensure reciprocity property

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- Therefore the proposed model is a visually plausible representation, since it only satisfies the reciprocity and non-negativity properties.

Table of Contents

1. Introduction
   - Definitions of BRDFs
   - Properties of Proposed BRDF Representation
   - Our Proposed BRDF Representation
2. Previous Works
3. Copula Distributions
   - Pair-Copula Constructions
4. BRDF Representation Using Pair-Copula Constructions
   - MIT MERL BRDF Dataset
   - Our BRDF Representation
   - Some Empirical Results
5. Our Estimation Procedure
6. Importance Sampling
7. Results
8. Conclusions and Future Work
9. Questions
10. Thank You
11. References

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Table of Contents

1. Introduction
   - Definitions of BRDFs
   - Properties of Proposed BRDF Representation
   - Our Proposed BRDF Representation
2. Previous Works
3. Copula Distributions
   - Pair-Copula Constructions
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   - Our BRDF Representation
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6. Importance Sampling
7. Results
8. Conclusions and Future Work
9. Questions
10. Thank You
11. References

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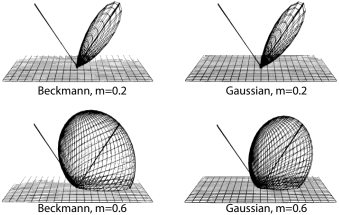

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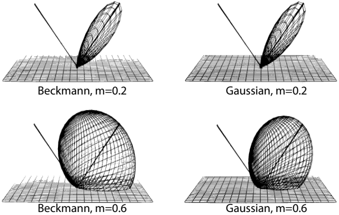

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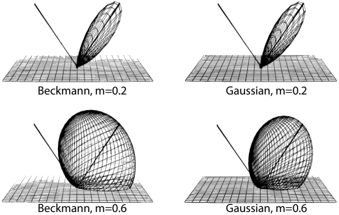

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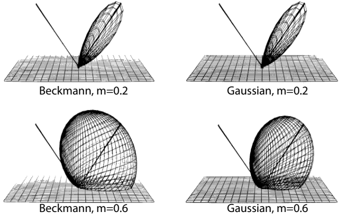

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  - Öztürk et al. [13] have modeled BRDF data using Archimedean copula distributions.

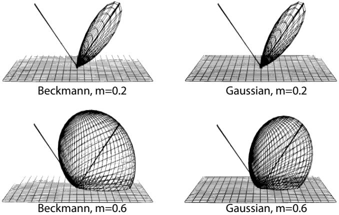

Table of Contents

1. Introduction
   - Definitions of BRDFs
   - Properties of Proposed BRDF Representation
   - Our Proposed BRDF Representation
2. Previous Works
3. Copula Distributions
   - Pair-Copula Constructions
4. BRDF Representation Using Pair-Copula Constructions
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   - Our BRDF Representation
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5. Our Estimation Procedure
6. Importance Sampling
7. Results
8. Conclusions and Future Work
9. Questions
10. Thank You
11. References

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- Joint density function f is given as:

$$f(x_1, x_2, ..., x_n) = c_{1\cdots n}\{F_1(x_1), F_2(x_2), ..., F_n(x_n)\} \prod_{i=1}^{n} f_i(x_i), \tag{4}$$

where $c_{1\cdots n}$ is the copula pdf and $f_i(x_i), i = 1, 2, ..., n$ are the marginal densities of joint pdf [7].

Table of Contents

1. Introduction
   - Definitions of BRDFs
   - Properties of Proposed BRDF Representation
   - Our Proposed BRDF Representation
2. Previous Works
3. Copula Distributions
   - Pair-Copula Constructions
4. BRDF Representation Using Pair-Copula Constructions
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   - Our BRDF Representation
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6. Importance Sampling
7. Results
8. Conclusions and Future Work
9. Questions
10. Thank You
11. References

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- For example, if a 3D pdf f with random variables X_1, X_2 and X_3 is given, and X_1, X_3 are independent given that X_2, then $c_{13|2}\{F(x_1|x_2), F(x_3|x_2)\} = 1$.

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- Thus, the joint pdf can be expressed as:

$$f(x_1, x_2, x_3) = c_{12}\{F_1(x_1), F_2(x_2)\}c_{23}\{F_2(x_2), F_3(x_3)\}\prod_{i=1}^{3} f_i(x_i). \tag{5}$$

where $u_i = F_i(x_i), i = 1, 2, ..., n$ are the marginal distributions of joint distribution F [10].

Table of Contents

1. Introduction
   - Definitions of BRDFs
   - Properties of Proposed BRDF Representation
   - Our Proposed BRDF Representation
2. Previous Works
3. Copula Distributions
   - Pair-Copula Constructions
4. BRDF Representation Using Pair-Copula Constructions
   - MIT MERL BRDF Dataset
   - Our BRDF Representation
   - Some Empirical Results
5. Our Estimation Procedure
6. Importance Sampling
7. Results
8. Conclusions and Future Work
9. Questions
10. Thank You
11. References

Table of Contents

1. Introduction
   - Definitions of BRDFs
   - Properties of Proposed BRDF Representation
   - Our Proposed BRDF Representation
2. Previous Works
3. Copula Distributions
   - Pair-Copula Constructions
4. BRDF Representation Using Pair-Copula Constructions
   - MIT MERL BRDF Dataset
   - Our BRDF Representation
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6. Importance Sampling
7. Results
8. Conclusions and Future Work
9. Questions
10. Thank You
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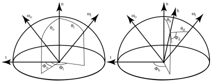

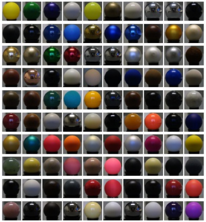

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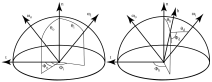

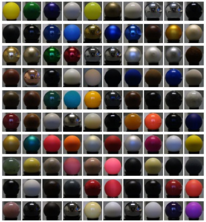

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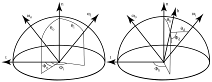

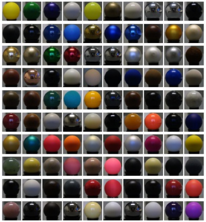

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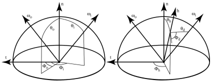
- The Rusinkiewicz parameterization depends on $\theta_h, \phi_h, \theta_d$ and ϕ_d. It is well-known that isotropic BRDF values are independent of ϕ_h.
- Therefore, the measured BRDF data of Matusik et al. [9] is sampled at $90, 90, 180$ resolutions for θ_h, θ_d and ϕ_d, respectively giving total of $90 \times 90 \times 180 = 1.458.000$ samples per color channel (Red-Green-Blue).



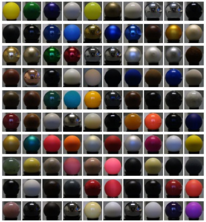


Table of Contents

1. Introduction
   - Definitions of BRDFs
   - Properties of Proposed BRDF Representation
   - Our Proposed BRDF Representation
2. Previous Works
3. Copula Distributions
   - Pair-Copula Constructions
4. BRDF Representation Using Pair-Copula Constructions
   - MIT MERL BRDF Dataset
   - Our BRDF Representation
   - Some Empirical Results
5. Our Estimation Procedure
6. Importance Sampling
7. Results
8. Conclusions and Future Work
9. Questions
10. Thank You
11. References

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- The scaled BRDF b_{ijk} is evaluated with the following expression:

$$b_{ijk} = \frac{b^*_{ijk}}{K}, \tag{6}$$

where b^*_{ijk} is the measured BRDF, and
$K = \delta_{\theta_h}\delta_{\theta_d}\delta_{\phi_d}\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{r} b^*_{ijk}$ is the scaling factor.

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- After the scaling transformation, the normalized BRDF, b_{ijk} can be modeled in terms of pair-copulae as:

$$\begin{aligned} b_{ijk} &= f_{\theta_h}(\theta_h^i) f_{\theta_d}(\theta_d^j) f_{\phi_d}(\phi_d^k) c_{\theta_h\theta_d}\{F_{\theta_h}(\theta_h^i), F_{\theta_d}(\theta_d^j)\} \\ & \quad c_{\theta_h\phi_d}\{F_{\theta_h}(\theta_h^i), F_{\phi_d}(\phi_d^k)\} \\ & \quad c_{\theta_d\phi_d|\theta_h}\{F(\theta_d^j|\theta_h^i), F(\phi_d^k|\theta_h^i)\}. \end{aligned} \tag{7}$$

Table of Contents

1 Introduction
- Definitions of BRDFs
- Properties of Proposed BRDF Representation
- Our Proposed BRDF Representation

2 Previous Works

3 Copula Distributions
- Pair-Copula Constructions

4 BRDF Representation Using Pair-Copula Constructions
- MIT MERL BRDF Dataset
- Our BRDF Representation
- Some Empirical Results

5 Our Estimation Procedure

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7 Results

8 Conclusions and Future Work

9 Questions

10 Thank You

11 References

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- It is seen from Figure 3 that highest fitting errors were observed for $\theta_h < 45^\circ$ for most of the materials. After the 45 degrees the distribution are nearly similar.

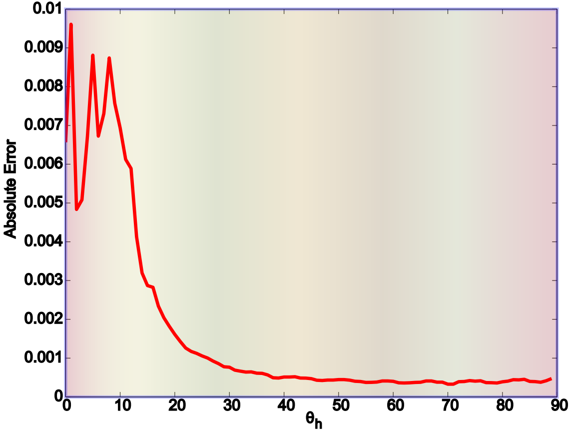

Figure 1 : Absolute fitting errors on every θ_h of measured dark-red-paint material (red channel).

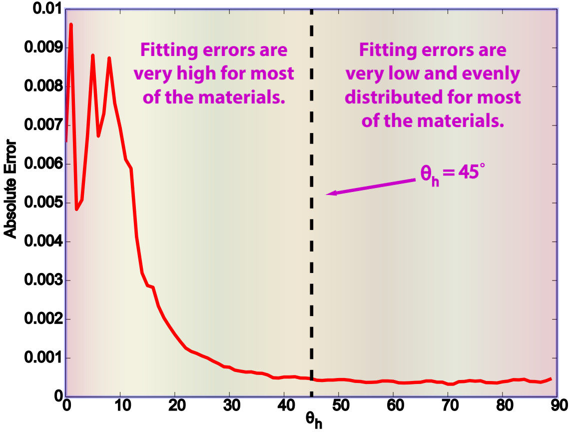

Figure 2 : Absolute fitting errors on every θ_h of measured dark-red-paint material (red channel).

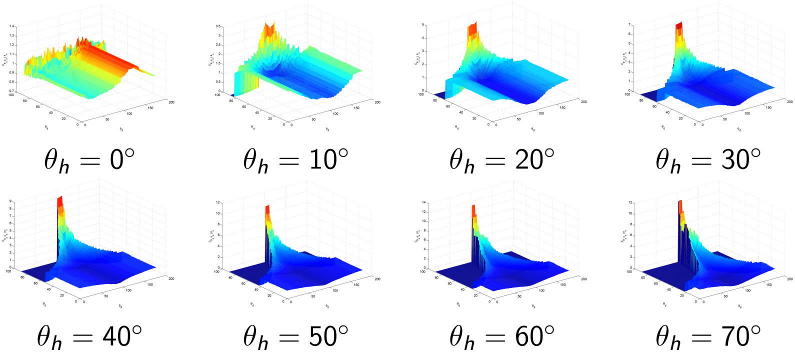

Figure 3 : 2D $c_{\theta_d \phi_d | \theta_h}$ distributions of measured dark-red-paint material for various θ_h angles (red channel).

Table of Contents

1. Introduction
   - Definitions of BRDFs
   - Properties of Proposed BRDF Representation
   - Our Proposed BRDF Representation
2. Previous Works
3. Copula Distributions
   - Pair-Copula Constructions
4. BRDF Representation Using Pair-Copula Constructions
   - MIT MERL BRDF Dataset
   - Our BRDF Representation
   - Some Empirical Results
5. Our Estimation Procedure
6. Importance Sampling
7. Results
8. Conclusions and Future Work
9. Questions
10. Thank You
11. References

Our Estimation Procedure

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5. To render a color image, we follow a similar approach that was used by Ngan et al. [11], and we estimate the diffuse and specular parameters for each pair of measured BRDF of each color channel and the approximate BRDF values using a robust linear regression procedure [5].

Table of Contents

1. Introduction
   - Definitions of BRDFs
   - Properties of Proposed BRDF Representation
   - Our Proposed BRDF Representation
2. Previous Works
3. Copula Distributions
   - Pair-Copula Constructions
4. BRDF Representation Using Pair-Copula Constructions
   - MIT MERL BRDF Dataset
   - Our BRDF Representation
   - Some Empirical Results
5. Our Estimation Procedure
6. **Importance Sampling**
7. Results
8. Conclusions and Future Work
9. Questions
10. Thank You
11. References

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- Then, we model ρ and p_o using pair-copula constructions and wavelet transforms. The computational cost of this sampling procedure is very expensive since generating incoming vectors from this 2D conditional pdf is not efficient.

Table of Contents

1 Introduction
- Definitions of BRDFs
- Properties of Proposed BRDF Representation
- Our Proposed BRDF Representation

2 Previous Works

3 Copula Distributions
- Pair-Copula Constructions

4 BRDF Representation Using Pair-Copula Constructions
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- Our BRDF Representation
- Some Empirical Results

5 Our Estimation Procedure

6 Importance Sampling

7 Results

8 Conclusions and Future Work

9 Questions

10 Thank You

11 References

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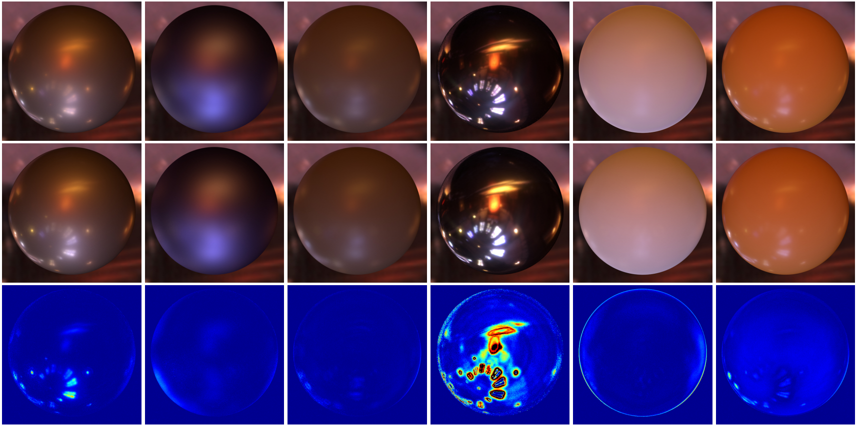
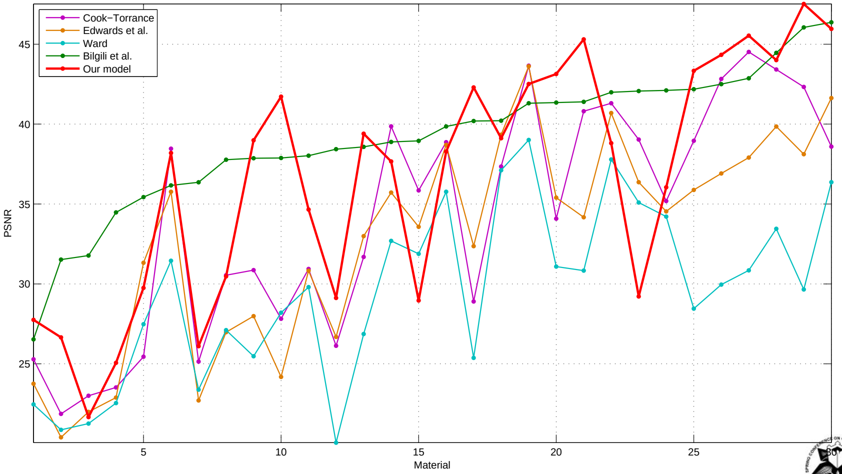
- Based on the data set [9] we used, we need to store 60.4 KB data for each material, which requires 33 MB storage space (1/600 compression).



(Alum-Bronze)	(BMP)	(Fruitwood-241)	(Chrome-Steel)	(Delrin)	(YMP)
$(PSNR = 39.63)$	$(PSNR = 43.16)$	$(PSNR = 45.74)$	$(PSNR = 26.77)$	$(PSNR = 39.38)$	$(PSNR = 37.66)$


Cook−Torrance
Edwards et al.
Ward
Bilgili et al.
Our model
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25
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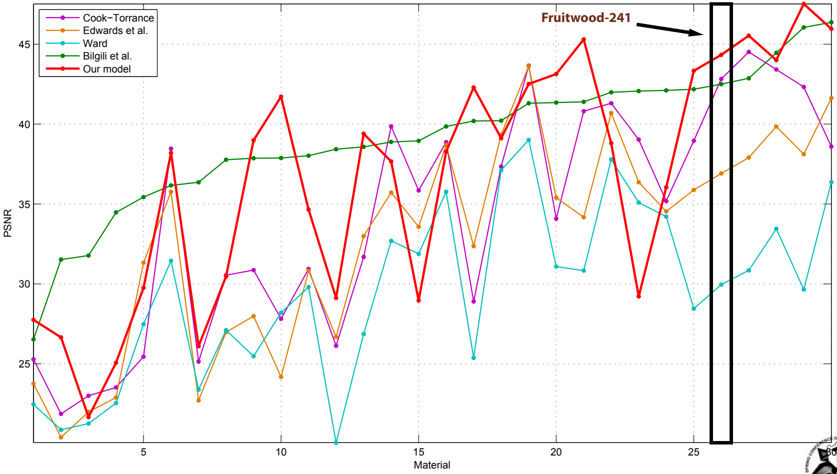
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Fruitwood-241
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Material
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30
35
40
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5
10
15
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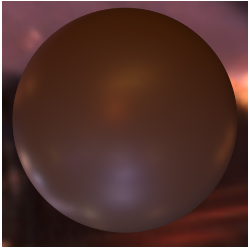
25
30

Fruitwood-241 Material



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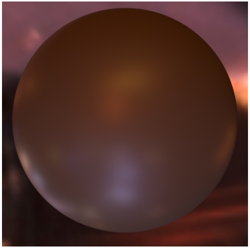
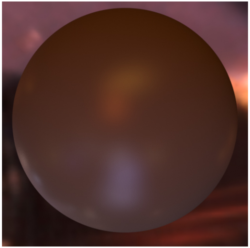
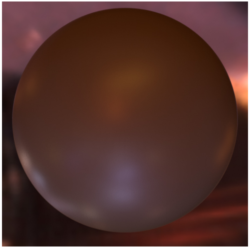


(Cook-Torrance, $PSNR = 42.83$)



(Edwards et al., $PSNR = 36.91$)



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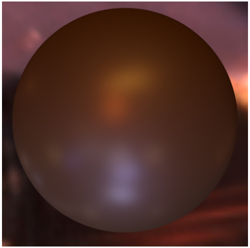
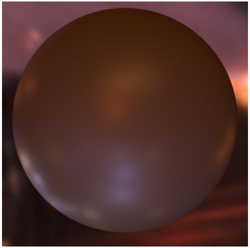
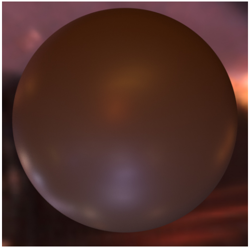


(Ward, $PSNR = 29.96$)



(Our Model, $PSNR = 44.34$)

Fruitwood-241 Material - Difference Images



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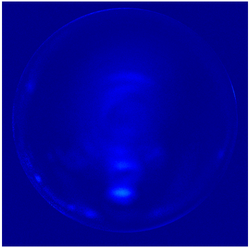

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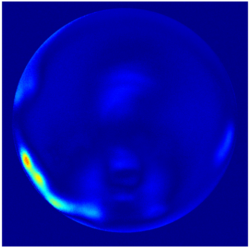
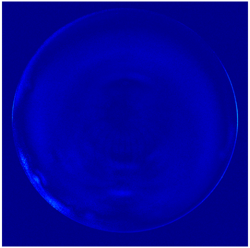


(Edwards et al., $PSNR = 36.91$)



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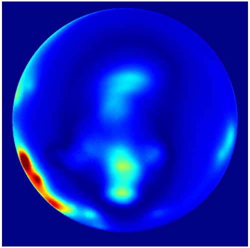

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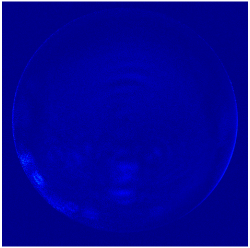

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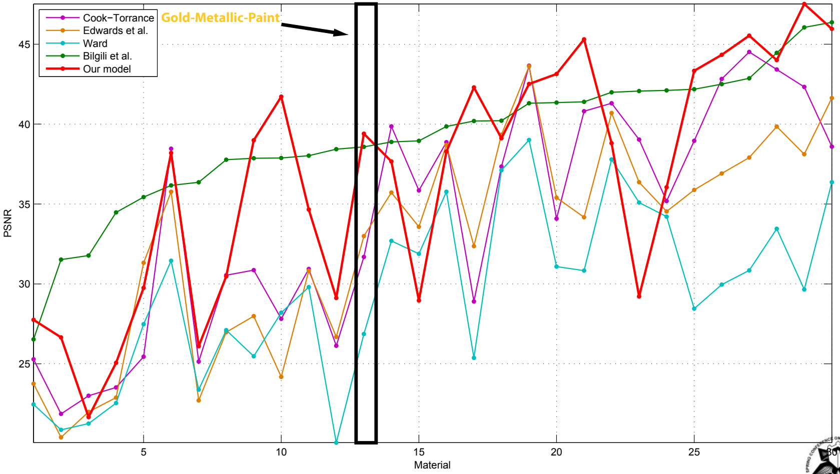
Cook−Torrance
Edwards et al.
Ward
Bilgili et al.
Our model
Gold-Metallic-Paint
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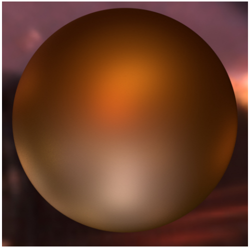
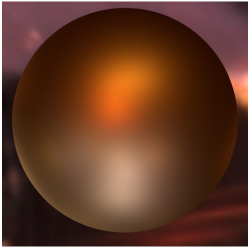
Gold-Metallic-Paint Material

(Reference)

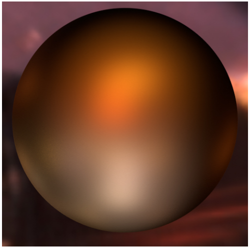


(Cook-Torrance, $PSNR = 31.69$)



(Edwards et al., $PSNR = 32.99$)

(Bilgili et al., $PSNR = 38.57$)

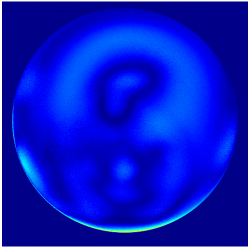


(Ward, $PSNR = 26.87$)

(Our Model, $PSNR = 39.4$)

Gold-Metallic-Paint Material - Difference Images

(Reference)



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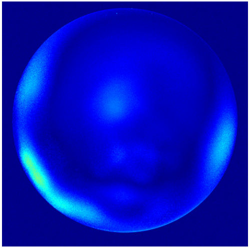

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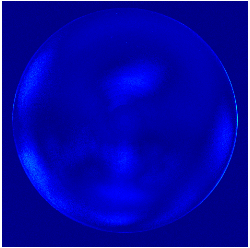

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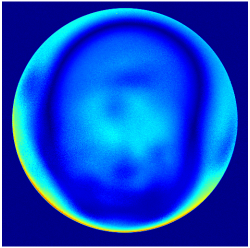

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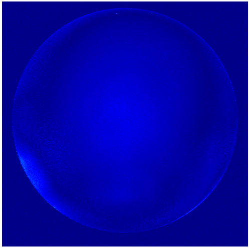

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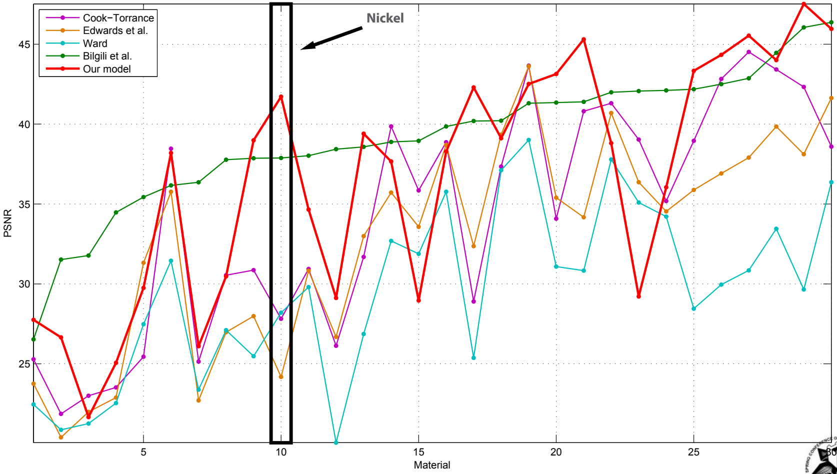
Cook−Torrance
Edwards et al.
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Our model
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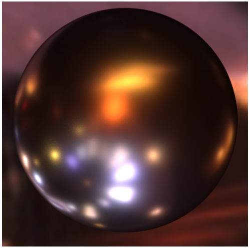
Nickel Material



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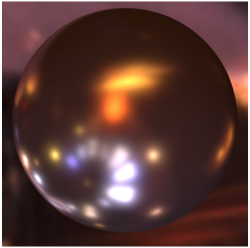
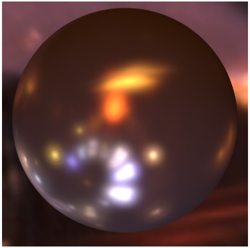
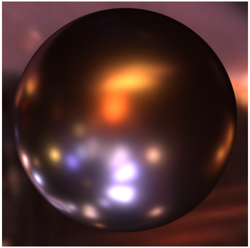


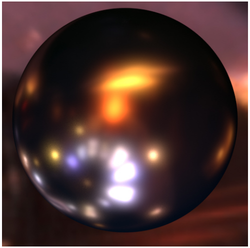
(Cook-Torrance, $PSNR = 27.82$)



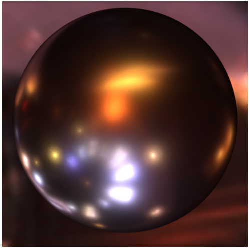
(Edwards et al., $PSNR = 24.19$)



(Bilgili et al., $PSNR = 37.88$)

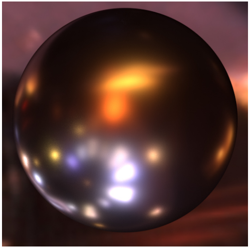


(Ward, $PSNR = 28.20$)



(Our Model, $PSNR = 41.72$)

Nickel Material - Difference Images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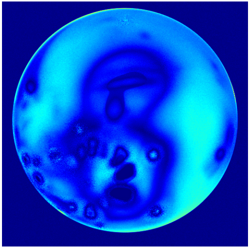
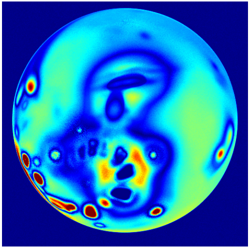

(Cook-Torrance, $PSNR = 27.82$)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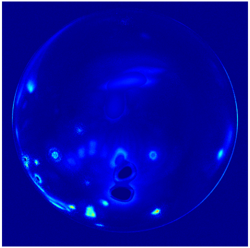
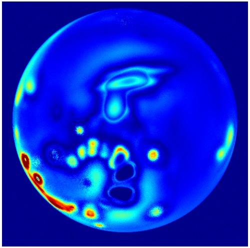

(Bilgili et al., $PSNR = 37.88$)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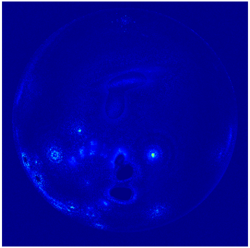
(Our Model, $PSNR = 41.72$)

Table of Contents

1. Introduction
   - Definitions of BRDFs
   - Properties of Proposed BRDF Representation
   - Our Proposed BRDF Representation
2. Previous Works
3. Copula Distributions
   - Pair-Copula Constructions
4. BRDF Representation Using Pair-Copula Constructions
   - MIT MERL BRDF Dataset
   - Our BRDF Representation
   - Some Empirical Results
5. Our Estimation Procedure
6. Importance Sampling
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8. Conclusions and Future Work
9. Questions
10. Thank You
11. References

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- In this paper, we introduced a compact technique to represent BRDF data using pair-copula constructions and wavelet transforms.
- Our technique also can be generalized for compression of data from any multivariate distribution.
- It is empirically shown that the proposed technique has provided satisfactory results.
- As a future work;
  - It can be shown that this compression technique can be generalized to higher dimensional problems such as Bidirectional Scattering Surface Reflectance Distribution Function (BSSRDF), Spatially Varying Bidirectional Reflectance Distribution Function (SVBRDF), Bidirectional Texture Function (BTF).

Conclusions and Future Work

- In this paper, we introduced a compact technique to represent BRDF data using pair-copula constructions and wavelet transforms.
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  - We also would like to represent 4D measured anisotropic BRDF data with our representation.

Table of Contents

1. Introduction
   - Definitions of BRDFs
   - Properties of Proposed BRDF Representation
   - Our Proposed BRDF Representation
2. Previous Works
3. Copula Distributions
   - Pair-Copula Constructions
4. BRDF Representation Using Pair-Copula Constructions
   - MIT MERL BRDF Dataset
   - Our BRDF Representation
   - Some Empirical Results
5. Our Estimation Procedure
6. Importance Sampling
7. Results
8. Conclusions and Future Work
9. Questions
10. Thank You
11. References

Questions

Table of Contents

1. Introduction
   - Definitions of BRDFs
   - Properties of Proposed BRDF Representation
   - Our Proposed BRDF Representation
2. Previous Works
3. Copula Distributions
   - Pair-Copula Constructions
4. BRDF Representation Using Pair-Copula Constructions
   - MIT MERL BRDF Dataset
   - Our BRDF Representation
   - Some Empirical Results
5. Our Estimation Procedure
6. Importance Sampling
7. Results
8. Conclusions and Future Work
9. Questions
10. Thank You
11. References

THANK YOU!

Table of Contents

1 Introduction
- Definitions of BRDFs
- Properties of Proposed BRDF Representation
- Our Proposed BRDF Representation

2 Previous Works

3 Copula Distributions
- Pair-Copula Constructions

4 BRDF Representation Using Pair-Copula Constructions
- MIT MERL BRDF Dataset
- Our BRDF Representation
- Some Empirical Results

5 Our Estimation Procedure

6 Importance Sampling

7 Results

8 Conclusions and Future Work

9 Questions

10 Thank You

11 References

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