# A Heterogeneous Subsurface Scattering Representation Based on Compact and Efficient Matrix Factorization

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**Abstract**

*This poster presents a novel compact and efficient factored subsurface scattering representation for heterogeneous translucent materials. Our subsurface scattering representation consists of two parts, namely, a matrix factorization and a linear regression method. We first apply a matrix factorization method on the intensity profiles of the heterogeneous subsurface scattering responses. Next, we fit a polynomial model for characterizing the differences between the different color channels with a linear regression procedure. We validate our heterogeneous subsurface scattering representation on various real-world heterogeneous translucent materials, geometries and lighting conditions. We show that our method provides good compression at acceptable visual accuracy.*

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Color, shading, shadowing, and texture

## 1. Introduction

Translucent materials, such as wax, and marble are common in real-world. They also have unique heterogeneous structures. To render such composite materials, acquiring heterogeneous data is a convenient way that yields realistic results [PvBM*06]. However, due to the enormous storage requirements (gigabytes), efficient compression algorithms are necessary. In this poster, we present our compact and efficient factorization based subsurface scattering representation, suitable for representing and rendering the spatial component of heterogeneous subsurface scattering materials.

## 2. Subsurface Scattering Representation

**Pre-Processing:** We represent the measured BSSRDF as a $2D$ matrix $R_d(x_i,x_o)$ where $x_i$ and $x_o$ are incoming and outgoing surface locations. We can get the most compact form of the subsurface scattering matrix $R_d(x_i,x_o)$ with $d = x_o - x_i$ reparametrization. The reparameterized subsurface scattering matrix $R'_d(x_i,d)$ can be factorized instead of $R_d(x_i,x_o)$. To increase the effectiveness of a classical factorization, we shift each row independently such that the maximum element in the each row will be the first element in that row. After that, we divide each row with its maximum value. After these operations, we get $R''_d(x_i,d)$ from the reparameterized subsurface scattering matrix $R'_d(x_i,d)$. In our experiments, we see that $R''_d(x_i,d)$ is more suitable for an efficient factorization than $R'_d(x_i,d)$. Another advantage of the shifting of the rows is that this also allows to efficiently compensate for any shift in the peak that can occur due to measurement or calibration issues.

**Factorization:** As we describe in the previous subsection, we prepare the reparameterized subsurface scattering matrix $R''_d(x_i,d)$ which includes intensity response values for each surface point. For an efficient and compact factorization, we apply the error modeling approach using the Tucker factorization [Tuc66] to $R''_d(x_i,d)$ matrix. Please refer to [BÖK11] for an in depth discussion on the error modeling approach. Consequently, our subsurface scattering model can be formalized as:

$$R''_d(x_i,d) \approx \sum_{j=1}^{T} g_j f_j(x_i) h_j(d), \qquad (1)$$

where $T$ is the total number of terms, $g_j$ is the scalar core tensor, $f_j(x_i)$ and $h_j(d)$ are the univariate tensor functions, $x_i$ incoming surface location and $d = x_o - x_i$. Since we apply Tucker factorization to $2D$ intensity matrix, our Tucker-based factorization algorithm is the same as a classical Singular Value Decomposition (SVD) method.

**Linear Regression:** In the linear regression procedure, we

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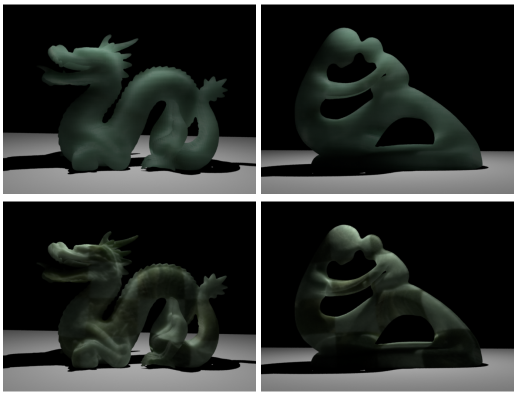

**Figure 1:** *Representing heterogeneous subsurface scattering with our factored model. (Upper) marble (close up) material ($T = 15, P = 4$), (bottom) chessboard ($4 \times 4$) material ($T = 15, P = 7$).*

estimate the linear coefficients for each row of measured subsurface scattering matrix. Then, the corresponding models for each color channel can be written as

$$R_{dr}(x_i, x_o) \approx \sum_{p=0}^{P} \beta_{rpx_i} R'_d(x_i, d)^p, \quad (2)$$

$$R_{dg}(x_i, x_o) \approx \sum_{p=0}^{P} \beta_{gpx_i} R'_d(x_i, d)^p, \quad (3)$$

$$R_{db}(x_i, x_o) \approx \sum_{p=0}^{P} \beta_{bpx_i} R'_d(x_i, d)^p, \quad (4)$$

where $P$ is the degree of the polynomial, and $\beta_{rpx_i}$, $\beta_{gpx_i}$, and $\beta_{bpx_i}$ are the parameters of the model. Linear least square optimization techniques were used to fit the model to subsurface scattering data and the subsurface scattering values for the underlying color channel were estimated from the fitted model. Our linear regression based method exploits coherency between the color channels, and provides a more compact representation without significant loss of accuracy. This approach can potentially also be applied other factorization based compression methods.

## 3. Results

To visualize our results, we implemented a rendering scheme similar to Peers et al. [PvBM*06] in the Mitsuba rendering system [Jak13]. We verified our Tucker factorization based subsurface scattering model on several real-world subsurface scattering materials, ranging from fairly homogeneous to highly translucent heterogeneous materials. As can be seen in Figure 1, our Tucker-based subsurface scattering model can be used with any geometries, while providing heterogeneous subsurface scattering effects visually plausibly. We also compared our model with Peers et al.'s [PvBM*06] subsurface scattering model. As can be seen in Figure 2, our Tucker factorization based subsurface scattering model represents heterogeneous translucent materials more accurately at the same compression rates.

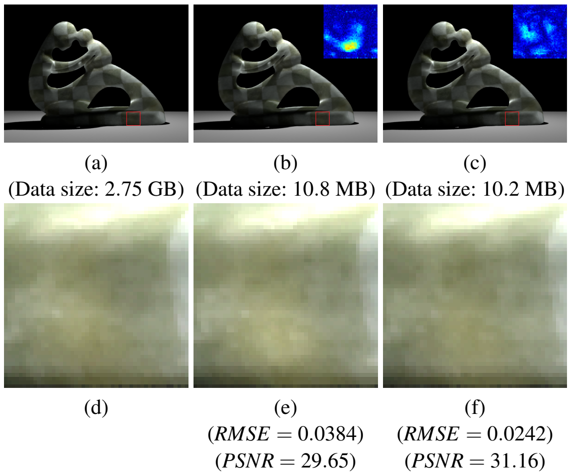


**Figure 2:** *For visual comparison on a statue under spot lighting, (a) a heterogeneous chessboard ($8 \times 8$) was rendered with a full Monte Carlo path tracing algorithm (reference image); (b) and (c) were rendered using Peers et al. [PvBM*06] and our factored subsurface scattering model, respectively. (d), (e), and (f) are zoom-in images from (a), (b), and (c), respectively. For better comparison, false-color differences were scaled by a factor of* 5.

## 4. Conclusions and Future Work

In this poster we have presented a compact and efficient factorization based representation for the spatial component of heterogeneous subsurface scattering. Our subsurface scattering representation is composed of Tucker factorization and a linear regression procedure. We have demonstrated that our compact factored representation can be applied to any geometries and it can be easily integrated into a standard global illumination rendering system, resulting in convincing images. Furthermore, we compared our subsurface scattering model with Peers et al.'s [PvBM*06] factored model, and we showed that our compact subsurface scattering model can represent heterogeneous subsurface scattering effects accurately. In the future, we are interested in exploring rendering algorithms to employ our subsurface scattering representation directly in real-time applications.

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